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14. ABSTRACT
Research accomplishments under ARO support can be grouped into seven problem areas. In *reliability economics*, we solved the important open problem of identifying optimal system designs relative to criterion functions based on both performance and cost. We also solved the related statistical estimation problem, allowing data-based approximations of optimal system designs. New results were obtained in *structural reliability*, and a research monograph in this area was completed. Sufficient conditions were obtained for the *optimal allocation* of the reliabilities p(1),…, p(n) to the n components of a coherent system. Nonparametric inference under type I censoring has been developed for the distribution of

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Report Title

Final Report on ARO grant W911NF-05-1-0118 entitled "Studies in Reliability Economics and Reliability Modeling and Inference"

ABSTRACT

Research accomplishments under ARO support can be grouped into seven problem areas. In *reliability economics*, we solved the important open problem of identifying optimal system designs relative to criterion functions based on both performance and cost. We also solved the related statistical estimation problem, allowing data-based approximations of optimal system designs. New results were obtained in *structural reliability*, and a research monograph in this area was completed. Sufficient conditions were obtained for the *optimal allocation* of the reliabilities p(1),…, p(n) to the n components of a coherent system. Nonparametric inference under type I censoring has been developed for the distribution of bug discovery times in *software reliability*. Work on *systems with weighted components* clarifies their place within the theory of coherent systems and sheds light on their monotonicity properties. We introduced the concept of "self-consistency" in *Bayesian inference* and demonstrated its relevance to the "consensus problem". New inference procedures for dealing with autopsy data from *stress-strength experiments* have been justified.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

[1] "Characterizations of the Relative Behavior of Two Systems via Properties of Their Signature Vectors", in Advances in Distribution Theory, Order Statistics and Inference, 279 - 290, (2006) (with H. Block and M. Dugas)

[2] "On Constrained Estimation from Time Use Survey Data", Statistics and Probability Letters, 77, 204-10 (2007) (with E. Vestrup and D. Bhattacharya)

[3] "Signature-Related Results on System Failure Rates and Lifetimes", in Advances in Statistical Modeling and Inference, V. Nair, Editor, Singapore: World Scientific, 115 – 30 (2007) (with H. Block and M. Dugas)

[4] "Estimation based on Autopsy Data from Stress-Strength Experiments", Journal of Quality Technology and Quality Management, Special Issue on Reliability, 4, 1-15 (2007)

[5] "On Optimal System Design in Reliability-Economics Frameworks", Naval Research Logistics, 54(5), 568 – 82, (2007) (with M. Dugas)

[6] "Imperfect Repair", in Encyclopedia on Statistics in Quality and Reliability, Ruggeri, F, Kenett, R. and Faltin, F., Editors, John Wiley & Sons Ltd, Chichester, UK, 843-847 (2007) (with M. Hollander and J. Sethuraman)

[7] "System Signatures", in Encyclopedia on Statistics in Quality and Reliability, Ruggeri, F, Kenett, R. and Faltin, F., Editors, John Wiley & Sons Ltd, Chichester, UK, 1982-1986 (2007)

[8] "Nonparametric Methods for Analysis of Repair Data", in Encyclopedia on Statistics in Quality and Reliability, Ruggeri, F, Kenett, R. and Faltin, F., Editors, John Wiley & Sons Ltd, Chichester, UK, 1248-1252 (2007) (with M. Hollander and J. Sethuraman)

[9]"Nonparametric Analysis of the Order–Statistic Model in Software Reliability", IEEE lTransactions on Software Reliability, 33, 198 – 208 (2007) (with S. Wilson)

[10] "On the Optimal Allocation of Components within Coherent Systems", Statistics and Probability Letters, 938 - 943 (2007) (with D. Bhattacharya)

[11] "Systems with Weighted Components", Statistics and Probability Letters, 78, 815 – 23 (2008) (with M. Shaked)

Number of Papers published in peer-reviewed journals: 11.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

[1] “On Optimal System Design under Reliability and Economic Constraints”, Proceedings: the 10th Army Conf. on Applied Statistics, MD: Aberdeen Proving Ground , (2005), (with M. Dugas)

[2] “On Conjugacy, Self-consistency and Consensus in Bayesian Inference”, Proceedings: the 11th Army Conference on Applied Statistics, MD: Aberdeen Proving Ground, (2006)

[3] “On Comparing Systems of Different Sizes”, Proceedings: the 12th Army Conference on Applied Statistics, MD: Aberdeen Proving Ground, (2007)

Number of Papers published in non peer-reviewed journals: 3.00

(c) Presentations

[1] "On the Optimal Allocation of Components within Coherent Systems", presented at the Army Conference on Applied Statistics, October, 2007

[2] "System Signatures, Order Statistics and Engineering Reliability", presented as an invited plenary lecture at the 8th International Conference on Ordered Statistical Data and Its Applications, Aachen, Germany, March, 2008

Number of Presentations: 2.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

[1] “On Comparing the Reliability of Arbitrary Systems via Stochastic Precedence”, Advances in Mathematical Modeling for Reliability, T. Bedford, J. Quigley, L. Walls, B. Alkali, A. Daneshkhah and G. Hardman, Editors, 129 – 137, (2008) IOS Press (with M. Hollander) -- based on a paper presented at the International Conference on Mathematical Methods in Reliability, July, 2007, Glasgow, Scotland.

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 1

(d) Manuscripts

[1] “Life Testing in a Weibull Environment”, submitted for publication (with Y. S. Chong)

[2] “On Hierarchical and Linear Bayes Approaches to Combining Data from ‘Related’ Experiments”, submitted for publication (with D. Steffey and H. Tran)

Number of Manuscripts: 2.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED
Michael McAssey	0.10
Michael Dugas	0.30
FTE Equivalent:	**0.40**
Total Number:	**2**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Bo Lindquist	0.05	No
FTE Equivalent:	**0.05**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

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The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
Michael R. Dugas	
Total Number:	**1**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

FINAL REPORT

1. **PERIOD COVERED BY REPORT: April 1, 2005 – June 30, 2008**
2. **PROPOSAL TITLE: "Studies in Reliability Economics and Reliability Modeling and Inference"**
3. **CONTRACT NUMBER: ARO Contract W911NF-05-1-0118**
4. **AUTHOR OF THE REPORT: Francisco J. Samaniego, PI**
5. **PERFORMING ORGANIZATION: University of California, Davis**

Our research during this period can be grouped into seven different problem areas. (1) The first is reliability economics. In this area, we have obtained solutions to certain optimization problems involving criterion functions based on both performance and cost. We have also solved the related statistical estimation problems that allow data-based approximations of optimal system designs. (2) New results have been obtained in the area of structural reliability, and a research monograph in this area has been completed. (3) Sufficient conditions have been obtained for a particular allocation of a set of reliabilities p(1),…, p(n) to the n components of a coherent system to be an optimal allocation, i.e., to maximize the system's reliability. (4) Nonparametric inference under type I censoring has been developed for the distribution of discovery times of bugs in the context of the assessment of software reliability. (5) Work on systems with weighted components has served to clarify their place within the theory of coherent systems, and has also shed light on some of their monotonicity properties. (6) We have also introduced the concept of "self-consistency" in Bayesian inference and demonstrated its relevance to the well-known "consensus problem" and (7) new inference procedures for dealing with autopsy data from stress-strength experiments have been proposed and studied. In the following paragraphs, the results obtained in each of these areas are described.

1. Reliability Economics

Reliability Economics is a field that can be defined as the collection of all problems in which there is tension between the performance of systems of interest and their cost. Given such a problem, the aim is to resolve the tension through an optimization process that identifies the system that maximizes some appropriate criterion function (e.g. expected lifetime per unit cost). In the present work, we have focused on coherent systems of n independent and identically distributed (i.i.d.) components and mixtures thereof, and we have characterized both a system's performance and cost as functions of the system's signature vector. For a given family of criterion functions, a variety of optimality results are obtained for systems of arbitrary order n. Approximations are developed and justified when the underlying component distribution is unknown. Assuming the availability of an auxiliary sample of N component failure times, the asymptotic theory of L-estimators is adapted for the purpose of establishing the consistency and asymptotic normality of the proposed estimators of the expected ordered failure times of the n components of the systems under study. These results lead to the identification of ε-optimal systems relative to the chosen criterion function. This work was published in Naval Research Logistics in 2007.

2. Structural Reliability

2a) Comparing systems of different sizes

Samaniego (1985) used system signatures to provide a characterization of systems that were IFR when their components are i.i.d. IFR. Kochar, Mukerjee and Samaniego (1999) proved that stochastic, hazard rate and likelihood ratio ordering between a pair of signatures guaranteed the same ordering between the corresponding system lifetimes. The definition of system signatures (and the theoretical results concerning them) has been shown to apply equally to mixed systems, that is, to stochastic mixtures of coherent systems. However, results obtained heretofore on the comparative analysis of systems apply only to systems of the same size. In our most recent work, a vehicle is developed which makes it possible to compare two mixed systems of arbitrary sizes. For any mixed system of size n based on components with i.i.d. lifetimes ~ F, it is shown that there is a system of size n+1 based on components with i.i.d. lifetimes ~ F that has the same lifetime distribution and is thus equivalent to the original system. Repeated applications of this result allows one to compare two systems of sizes $n < m$ by identifying a system of size m that is equivalent to the smaller system and comparing it to the larger system using existing tools. Several examples of such comparisons are given. This work was presented at the 12th Army Conference on Applied Statistics in October, 2006, and has been published in the proceedings of that conference.

2b) Comparing systems via stochastic precedence

While various forms of stochastic domination (including stochastic, hazard rate or likelihood ratio ordering) of one random variable over another have proven useful in making comparisons between systems, they share a common limitation. These modes of comparing systems induce only a partial ordering on the class of systems of interest, leaving some pairs of systems non-comparable. Comparisons via stochastic precedence, as defined in Arcones, Kvam and Samaniego (2002), do not have this limitation. In this investigation, we describe how stochastic precedence may be used as a metric in comparing arbitrary systems whose components are assumed to be independent and identically distributed. An explicit computational formula is obtained for the relevant probability $P(T(1) \leq T(2))$, where $T(1)$ and $T(2)$ are system lifetimes, in the case where the components of the systems in question have i.i.d. lifetimes. A necessary and sufficient condition depending solely on system signatures may be given for stochastic precedence between system lifetimes. Examples of pairs of systems whose lifetimes are not comparable by stochastic, hazard rate or likelihood ratio ordering are given; it is shown then that definitive comparisons are possible via stochastic precedence. Several special results are obtained for comparisons between systems whose signatures are symmetric. This work was presented at the Conference on Mathematical Methods in Reliability in Scotland in July, 2007, and is published in the refereed proceedings of that conference.

2c) Monograph on signatures

During this reporting period, the PI completed work on a monograph entitled "System Signatures and their Application to Engineering Reliability". The monograph was published by Springer in the fall of 2007. The monograph gives a comprehensive overview of system signatures, describing them as a new and useful tool for the computation and comparison of system reliabilities. After providing the necessary background in structural reliability, including the treatment of structure functions, coherent systems, cut sets and pat sets, system signatures are defined and motivated. For systems in i.i.d. components, representation theorems are proven for the system's survival function, density function and failure rate. Preservation theorems are established showing that various ordering properties of signature vectors are inherited by the lifetimes of the corresponding systems. Applications considered include the treatment of consecutive k-out-of-n systems, direct and indirect majority systems, the closure of systems in i.i.d. IFR components and the comparison of different forms of redundancy. The notion of signatures is extended to communication networks, and the utility of signatures in comparing the reliability of such networks is demonstrated. Applications of signatures in optimization problems in Reliability Economics are also described. The monograph presents an up-to-date account of the theory and applications of system signatures, including a number of recent research findings described elsewhere in this report.

3. Optimal Allocation Problems

This work addresses the problem of determining the optimal arrangement of independent components with varying individual reliabilities to specific locations within a given coherent system in order to achieve the highest possible system reliability. Using the notion of criticality introduced by Boland, Proschan and Sethuraman (1989), sufficient conditions are specified under which a particular allocation of the individual reliabilities will be optimal. It is then noted that this theorem may be adapted to apply to any partial ordering of component criticalities, thereby yielding a corollary that may be used in tandem with algebraic arguments to identify the optimal allocation of component reliabilities in systems which fail to satisfy the sufficient conditions for optimality in the main result. This work, joint with D. Bhattacharya, has been published in *Statistics and Probability Letters*.

4. Software Reliability

In the literature of statistical inference in software reliability, the assumption of parametric models and random sampling of bugs has been pervasive. In reality, both assumptions are problematic, the first because of robustness concerns and the second due to logical and practical difficulties. In joint research with Simon Wilson of Trinity College, a new approach is taken which eschews parametric assumptions within an order-statistic paradigm. The objective of the work is to estimate, from data on discovery times observed within a type I censoring framework, both the underlying distribution F from which the discovery times are generated and N, the unknown number of bugs in the software. The resulting estimators are used to predict the time to the next failure. The approach utilizes methods and models of Bayesian nonparametric inference (more specifically, the Beta-Stacy process) and is applied to data generated from the Naval Tactical Data System (NTDS) and is also illustrated in various simulations. This work was published in March 2007 in the *IEEE Transactions on Software Reliability*.

5. Systems with Weighted Components

An interesting formulation of component inequality was introduced by Wu and Chen (1994). The formulation retains the standard stochastic approach to the analysis of system performance, but adds the possibility that the components are differently weighted, each thereby having a potentially different impact on the functioning or failure of the system involved. Wu and Chen (1994) christened the new concept "weighted k-out-of-n systems". In the present work, we (i.e., Samaniego and Shaked) have adopted the more general name "systems with weighted components" or SWC's, as this more aptly describes this particular family of systems.

An SWC may be formally defined as follows. Consider a system with n components, and suppose that, for $i = 1, 2, \ldots, n$, component i is associated with a weight $w(i) > 0$. The system is working as long as the sum of the weights of the failed components is less than or equal to a certain threshold $T > 0$; the system fails when this sum exceeds T. For a given system, the vector $\mathbf{w} = (w(1), w(2), \ldots, w(n))$ and the threshold T are fixed.

The goal of our investigation has been to clarify some of the basic characteristics of such systems. Our research on SWCs establishes some basic facts about such systems. Our main findings are that (i) every SWC of order n (that is, with n components) is equivalent to a coherent system (that is, a monotone system in which every component is relevant) of order less than or equal to n, (ii) the converse of this result is not true (that is, there are coherent systems that cannot be represented as SWCs), and (iii) conditions are given for a coherent system to have an SWC equivalent. Further, we study how the component weights influence the lifetime of the SWC. We also examine the reliability importance and structural importance of the components of an SWC and confirm that these increase with the weights. We also outline the process of determining the coherent system that is equivalent to a given linear or circular consecutive SWC. Finally, the implications of our findings on the computation of the reliability of an SWC are discussed. This work is described in detail in Samaniego and Shaked (2007).

6. On Conjugacy, Consensus and Self Consistency in Bayesian Inference

Until fairly recently, conjugate prior distributions served as essential tools in the implementation of Bayesian inference. Today, they occupy a much less prominent place in Bayesian theory and practice. In the present paper, we discuss the reasons for this, and argue that the devaluation of the role of conjugacy may be somewhat premature. To assist in this argument, we introduce a Bayesian version of the notion of "self consistency" in the context of point estimation, relative to squared error loss, of the parameter ϑ of an exponential family of distributions. In this setting, a prior distribution π with mean ϑ^* (or the corresponding Bayes estimator $\hat{\vartheta}_\pi$) is said to be self consistent (SC) if the equation $E(\vartheta \mid \hat{\vartheta} = \vartheta^*) = \vartheta^*$ is satisfied, where $\hat{\vartheta}$ is assumed to be a sufficient and unbiased estimator of ϑ. The SC condition simply states that if your experimental outcome agrees with your prior opinion about the parameter, then the experiment should not change your opinion about it. Surprisingly, many prior distributions do not enjoy this property. We will study self consistency and its extended form (the estimator $T(\hat{\vartheta})$ of ϑ is generalized SC relative to a prior π with mean ϑ^* if $T(\vartheta^*) = \vartheta^*$). The problem of estimating ϑ based on the prior opinion received from k experts is examined, and the properties of a particular class of "consensus estimators" are studied. Conditions ensuring generalized self-consistency and an important convexity property of these estimators are identified. We conclude by applying Samaniego and Reneau's (1994) results to generalized self consistent consensus estimators, characterizing the circumstances in which such estimators outperform classical procedures. This work was presented at the army Conference on Applied Statistics in 2006.

7. Inference based on Autopsy Data from Stress-Strength Experiments

If Y is a random variable representing the breaking strength of a given material and X is a random variable measuring the stress placed on that material, then the probability that the material will survive the stress to which it is subjected is simply $P = P(X < Y)$. In most applications involving stress-strength testing, the variables X and Y are modeled as independent with respective cumulative distribution functions G and F. Our investigation focuses on a problem that is new yet of substantial practical interest, that of estimating stress and strength distributions from autopsy data, for example, from data on welded steel bars from a collapsed bridge. The general framework studied involves a random pair (Y,Z), where Y is a real-valued random variable associated with the strength of randomly selected material and, given Y =y, Z is a Bernoulli variable with probability p = p(y), interpreted as the conditional probability of surviving a random stress when Y = y. We explore the situation in which inference concerning the strength distribution F is of interest, but the random variable Y cannot be observed directly. In such situations, one would seek to draw inferences concerning F from the observed values of Z. In a specific modeling scenario, we treat both classical and Bayesian versions of the problem of estimating model parameters, when identifiable, and develop a Bayesian treatment of the estimation of G and F in the presence of nonidentifiability. This work was published in *Quality Technology and Quantitative Management* in March, 2007.

8. References

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Wilson, S. and Samaniego, F. J. (2007) Nonparametric Analysis of the Order-Statistic Model in Software Reliability, *IEEE Transactions on Software Reliability*, 33, 198 – 208.

Wu, J.S. and Chen, R. J. (1994) An Algorithm for Computing the Reliability of Weighted k-out-of-n Systems, *IEEE Transactions on Reliability*, **43**, 327 – 28.